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选择代表机构：一种公理化视角 *

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Abstract

As the world's democratic institutions are challenged by dissatisfied citizens, political scientists and also computer scientists have proposed and analyzed various (innovative) methods to select representative bodies, a crucial task in every democracy. However, a unified framework to analyze and compare different selection mechanisms is missing, resulting in very few comparative works. To address this gap, we advocate employing concepts and tools from computational social choice in order to devise a model in which different selection mechanisms can be formalized. Such a model would allow for desirable representation axioms to be conceptualized and evaluated. We make the first step in this direction by proposing a unifying mathematical formulation of different selection mechanisms as well as various social-choice-inspired axioms such as proportionality and monotonicity.

1 引言

人们常常认为，代议制民主正处于危机之中（例如，参见Landemore（2020）一书中的第2章及其中的参考文献）。尤其是，每当代表机构作出的决定似乎违背了其应当代表的民众的利益时，人们就会质疑代表机构存在的正当性。与此相一致，皮尤研究中心开展的一项调查（Wike和Fetterolf，2021）发现，尽管全球仍然广泛支持代议制民主，但人们也强烈认为，现有的政治制度需要改革。

In this paper, we focus on the task of selecting a representative body, which is a crucial ingredient of all democratic institutions as argued by political scientists (Mill 1924; Mansbridge 2009; Landa and Pevnick 2020; Rae 1967; Lijphart 1990; Michener, Amorim Neto, and Civitarese 2021). There is no shortage of innovative proposals to change how representative bodies are selected around the world. For example, some propose to select representatives at random (a.k.a. *sortition*) (Bouricius 2020), to elect them through transitive delegations (a.k.a. *liquid democracy*) (Valsangiacomo 2021), or to drastically increase the size of parliaments (see, e.g., https://thirty-thousand.org). Each proposed method has its benefits and drawbacks; however, we lack a systematic way to evaluate and compare them. Specifically, while there are numerous works in computer science and political science analyzing the strengths and weaknesses of specific methods, principled comparisons are rare.[1]

我们呼吁发展一个统一框架，以更具原则性的基础来制定和比较创新的与传统的选拔机制。在同一框架内制定不同的机制，进而也使得在同一框架内制定不同的期望目标成为可能。尽管我们认为，从不同视角进行比较是可能的，而且事实上迫切需要这样做，但我们提出了一种关于选拔机制的公理化视角，其灵感来源于社会选择领域关于投票规则的丰富文献。在本文的主体部分，我们给出一个具体示例，展示如何从公理化视角对选拔机制进行系统比较。首先，我们提出一个简单而丰富的数学框架，用于制定不同的选拔机制。其次，我们定义了各种公理，以刻画合理代表性的概念。我们希望，这些公理能够用于定量研究民主创新核心中固有的、尚未得到充分理解的权衡。值得注意的是，我们研究计划的重点并不在于寻找"理想的"代表制度。相反，我们设想构建一个导航器，将选拔机制与公理对应起来。我们主张构建一幅连贯的图景，呈现相互竞争的选拔提案各自的优缺点，从而引导公共讨论关注社会正在面临何种权衡，而不是继续基于彼此割裂的理由，为相互竞争的选拔机制进行争论。

近年来，计算机科学与民主创新日益交织，计算机科学家正在解决不同代表制方案中出现的诸多算法设计与可扩展性问题，并以公理化方式分析这些方案。

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[1]现有的少数例外主要关注认知层面、代表性的稳健性以及多数一致性（Alouf-Heffetz 等，2022；Green-Armytage，2015；Gelauff 等，2022；Abramowitz 和 Mattei，2019）。

事实上，正是随着信息技术的发展，更加复杂的交互式投票模型才逐渐变得普遍（Brill 2018）。例如，Miller（1969）和 Tullock（1992）认为，得益于技术进步，近年来在全国范围内开展更为丰富的政治决策过程已成为可能。我们设想的研究计划同样依赖于计算机科学家的专业知识。更具体地说，AAMAS 社群中的许多领域都可以为我们的工作作出贡献，帮助我们从数学上制定并分析选拔机制及其期望标准。例如，（i）社会选择与合作博弈论社群擅长对投票规则进行公理化分析；（ii）协调、组织、制度与规范社群可以提供规范性视角；（iii）人类与人工智能／人机交互领域则可以帮助分析可用性方面的问题。

In turn, making progress on representative selection mechanisms has also direct benefits for their applications in computer systems. For example, some blockchains select validators via a *nominated proof-of-stake* protocol, and the representativeness of the selection is essential for the security of the system (Cevallos and Stewart 2021). Further afield, blockchain-enabled Decentralized Autonomous Organizations (DAOs) are at the forefront of testing innovative governance systems based on interactive procedures (Kashyap and Jeyasekar 2020; Anshelevich et al. 2021; Zhang and Zhou 2017). Good (e-)governance remains a vast open question (Hassan and De Filippi 2021).

2 Mathematical Framework

In this section, we outline a mathematical framework to model selection mechanisms. First, we define some preliminary notation. A matrix is stochastic if each row sums up to 1. For a natural number $n \in \mathbb{N}$, let $[n]$ denote the set $\{1, 2, \ldots, n\}$ and let $e_n \in \mathbb{N}^{1\times n}$ denote the row vector containing all ones. For a vector $a \in \mathbb{R}^{1\times n}$, let $\|a\|_1$ denote the ℓ_1-norm of a, i.e., $\|a\|_1 = \sum_{i=1}^{n} |a_i|$.

表征建模

我们为以下任务提出一个数学框架：群体$\mathcal{N} = [n]$由n名代理人组成，希望通过选择机制M选出$\mathcal{N}$的一个子集，作为代表机构。我们还假设，被选为代表机构成员的代理人可以拥有不同的投票权重，也就是说，在代表机构作出的决策中，一些代理人的投票比其他代理人的投票更具分量。形式上，给定$\mathcal{N}$，我们希望选出一个权重向量$\mathbf{w} \in \mathbb{R}^n_{\geq 0}$。对于每个$i \in \mathcal{N}$，如果$\mathbf{w}_i > 0$，则$i$将作为代表机构的一员当选，并拥有投票权重$\mathbf{w}_i$。由此产生的代表机构规模为$|\{i \in \mathcal{N} \mid \mathbf{w}_i > 0\}|$。

Representation Matrix. The relation of agents is captured by a *representation matrix* $\Gamma \in \mathbb{R}^{n\times n}$, where the entry Γ_{ij} describes how well agent j can represent agent i. Γ is a stochastic matrix that allows fractional entries to account for the fact that agent i may be best represented by a mixture of other agents. How well i feels represented by j may be based on complex interactions of multiple aspects such that the issues i cares about, the relative preferences of j and i on these and other issues, intrinsic characteristics of i and j, and the underlying social network capturing who knows who.[2] Hence, the matrix Γ captures the complex nature of potential representation between agents in a simple yet rich form.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2/3 & 1/3 & 0 & 0 & 0 \\ 0 & 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 2/5 & 1/5 & 2/5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

图 1：示例 1 中所述实例的代表矩阵 Γ。行和列均以代理人为索引。

示例1。考虑以下示例：$\mathcal{N}$ $=\{A, B, C, D, E\}$，其中 A 和 B 属于某个政党，而 C、 D 和 E 属于另一个政党。设想 A 和 E是各自党-派中寻求权力的极端候选人。此外， B 和 D完全具有党派性，绝不会希望由其党- 派之外的人代表。相比之下， C 的信念较为温和，也可以由其他持有非极端观点的候选人代表。这种情况可以用图 1 中给出的代表矩阵 Γ 表示。

Interpreting the Representation Matrix. The representation matrix can be interpreted as giving rise to voting behavior. Specifically, assuming that each agent can split their vote in an arbitrary way, the matrix entries can be thought of as the ideal split of an agent's vote into fractional votes. In this paper, we will focus on uninominal ballots, i.e., each agent can vote for exactly one other agent to be part of the representative body. Accordingly, we interpret the entry Γ_{ij} as the probability that agent i selects (i.e., votes for) agent j.[3]

Expected Vote Share. Using the probabilistic interpretation of Γ for uninominal ballots allows us to reason about the expected share of votes an agent receives. For this, let V_j be the random variable representing the vote share agent j receives under the representation matrix Γ. Let V be the vector of the n random variables $V_1, \ldots, V_n$. Then, the expected vote share $\mathbb{E}[V_j]$ of agent j is the sum of the j-th column of Γ, i.e., $\mathbb{E}[V_j] = \sum_{i=1}^{n} \Gamma_{ij}$ and $\mathbb{E}[V] = \Gamma^T e_n$. In an idealistic setting, we would select all agents as members of the representative body and give each agent j a voting weight of $\mathbb{E}[V_j]$, i.e., $\mathbf{w}_j = \mathbb{E}[V_j]$ for all $j \in \mathcal{N}$. Accordingly, to evaluate the quality of different selection mechanisms, we will

[2] In line with this reasoning, political theorists have argued that quality of representation is multidimensional and depends on different factors such as similarities between the representative and the constituents (*descriptive representation* (Mansbridge 1999)), alignment of interests and values (*gyroscopic representation* (Mansbridge 2009)), or advancement of constituents' interests by the representative (*substantive representation* (Pitkin 1967)).

[3] The translation of Γ to votes can be extended to other ballot formats such as approval ballots or ranked ballots.

将（理想的）预期得票份额 $\mathbb{E}[V]$ 与机制返回的代理人的投票权重进行比较，假设代理人按照 Γ所述进行投票。对于图 1 中给出的代表矩阵，代理人的预期得票份额向量为$\mathbb{E}[V] = \left(\frac{5}{3},\ \frac{23}{15},\ \frac{16}{15},\ 1,\ \frac{7}{5}\right)$。

代表矩阵的性质。我们设想，给定代表矩阵 Γ 的代数性质可以刻画与公理分析相关的 $\mathcal{N}$ 显著社会特征：极化群体可由分块矩阵 Γ来表征； Γ的迹的相对大小可量化群体中追求权力的代理人数量； Γ 的秩可反映代理人彼此之间的相关程度，等等。与社会选择理论中的公理化结果（Arrow、Sen 和 Suzumura，2002；Gaertner，2001）一致，我们预期能够找到一些不可能性定理，并通过限制 Γ的结构来规避这些定理。

选拔机制

我们希望分析选拔机制 M：这些机制以代理人的单选投票为输入，并返回一个权重向量。作为额外输入的一部分，我们的机制还可以接受一个预先指定的代理人子集 $\mathcal{C} \subseteq \mathcal{N}$ 作为候选人，以及一个整数 k，用于描述可被选入代表机构的代理人数量。对于机制 M 和议事团规模 k，我们定义一个函数 f^{M_k}：给定 Γ 和 $\mathcal{C}$，该函数返回候选人在 Γ 和 $\mathcal{C}$下的预期投票权重。[4] 形式上， f^{M_k} 是一个函数

$$f^{M_k} : \mathbb{R}^{n\times n} \times (2^{\mathcal{N}} \setminus \emptyset) \to \mathbb{R}^{n\times 1}$$

such that $\{i \in \mathcal{N} \mid f^{M_k}(\Gamma, \mathcal{C})_i > 0\}$ is a subset of C of size at most k. Here, $f^{M_k}(\Gamma, \mathcal{C})_i$ is the expected voting weight ($\mathbb{E}[\mathbf{w}_i]$) of candidate $i \in \mathcal{C}$ in a body of size k selected by mechanism M assuming that $i \in \mathcal{N}$ votes for $j \in \mathcal{C}$ with probability depending on Γ_{ij}.

We now describe how the expected voting weights for different selection mechanisms can be computed, using the representation matrix given in Figure 1 as a running example.

Direct Democracy (D) In direct democracy assemblies, all agents are elected in the represented body. Thus, $\mathcal{C} = \mathcal{N}$ and $f^D(\Gamma, \mathcal{N}) = e_n$ for all representation matrices Γ.

First-Past-The-Post (F) First-past-the-post voting is widely used around the world but also widely criticized for, among other things, leaving voters feeling underrepresented (Bogdanor 1997). In first-past-the-post, a voting weight of 1 is given to the candidate receiving the highest number of votes and 0 to all other candidates. Notably, in first-past-the-post elections the electorate is typically partitioned into different voting districts, each selecting its own representative. We focus on the single-district case; however, our model can be extended to parallel independent districts.

继续示例 1，设 $\mathcal{C} = \{A, B, C, E\}$。注意，函数 f^{F_1} 会调整代表矩阵，以考虑候选人集合：代理人只能投票给候选人，并且我们假设候选人总是选择自己。在这个例子中，投影到候选人集合上的代表矩阵变为

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}.$$

以 0.5 的概率， C 获得 2 票，而 A、B 和 E各获得 1 票（结果是 C 在代表机构中拥有投票权重 1，而所有其他代理人的投票权重均为零）；以 0.5 的概率， E 获得 2 票，而 A、B 和 C 各获得 1 票。因此，我们得到f^{F_1}（Γ、$\mathcal{C}$）$=$（0, 0、$1/2$、0、$1/2$）^{T}。

代理投票（P）在代理投票中，所有代理人都会获得一个预先定义的候选人池，每位代理人都可以将自己的投票权委托给其中一名候选人。所有候选人实际上都是代表机构的一部分，候选人的投票权与委托给他们的票数成正比。[5] 代理投票既在计算机科学领域（Cohensius and Meir 2017；Anshelevich et al. 2021）受到研究，也在政治学领域（Miller 1969）受到研究；一些研究还将其扩展到更加灵活的、基于议题的委托（Abramowitz and Mattei 2019）。

The expected voting weight under proxy voting is the sum of expected delegations for the proxies (as in First-Past-The-Post, we adapt the representation matrix to account for the candidate set). Assuming again $\mathcal{C} = \{A, B, C, E\}$ in Example 1, since D's vote goes to C with probability 0.5 and to E with probability 0.5, we get $f^P(\Gamma, \mathcal{C}) = (1, 1, 3/2, 0, 3/2)^T$.

Liquid Democracy (L) In liquid democracy, each agent can choose to be part of the representative body or delegate their vote to another agent. Delegations are transitive, i.e., if A delegates to B and B delegates to C, and C decides to be in the representative body, then C votes on behalf of themself, as well as A and B. The representative body consists of all agents who self-select, with their voting power being set to the number of votes (transitively) delegated to them plus one. Liquid democracy has received considerable attention in the computer science community by studying it from a procedural and epistemic perspective (Kahng, Mackenzie, and Procaccia 2021; Escoffier, Gilbert, and Pass-Lanneau 2019; Bloembergen, Grossi, and Lackner 2019; Zhang and Grossi 2021; Halpern et al. 2021; Brill et al. 2022; Gölz et al. 2021; Revel et al. 2022), developing dedicated supporting software (Behrens et al. 2014; Paulin 2020), and examining

[4]有些机制既不需要候选人集合 $\mathcal{C}$，也不需要代表机构的规模 k 作为输入（在这种情况下，我们从 f^{M_k}中删除 k）。

[5]Proxy voting is closely related to the widespread practice of *party-list elections* (Pukelsheim 2014), where agents vote for parties and the seats in the representative body are distributed so that the number of seats of a party is proportional to the number of received votes. We focus on proxy voting as it allows for a cleaner mathematical formulation.

possible extensions (Ford 2002; Colley, Grandi, and Novaro 2022; Brill et al. 2022). Liquid Democracy has also been scrutinized from a political science perspective (Miller 1969; Blum and Zuber 2016; Valsangiacomo 2022).

为了找出传递委托下代理人的预期投票权重，我们利用代表矩阵穷尽所有可能的传递委托配置并计算其概率。例如，示例 1 中的一种可能配置是每个代理人都为自己投票（发生概率为 $_{2\,45}$），导致所有代理人都成为代表机构的一部分，并且投票权重为 1。[6] 总体而言，预期投票权重如下：$f^L(\Gamma,\mathcal{N}) = \left(\frac{89}{45},\ _{22\,45},\ _{14\,15},\ _{1\,5},\ \frac{7}{5}\right)^T$。

抽签遴选 (S) 抽签遴选是一种选择方法，它随机抽取人口的一个子集作为代表机构（Mueller、Tollison 和 Willett 1972；Dowlen 2017；Landemore 2020；Flanigan 等人 2020）。该方法允许平等参与决策，且不需要投票阶段。尽管被选中但不参与代表机构的代理人会对抽签遴选提供的公平性保证造成问题。计算机科学家正在研究处理此问题的算法方法（Flanigan 等人 2020、2021）。要找到的代表机构具有固定大小 $k < n$，并从 $\mathcal{N}$ 中均匀随机找到。代表机构的所有成员具有相等的投票权重。因此，每个代理人的预期投票权重为 $\frac{k}{n}$，即 $f^{S_k}(\Gamma,\mathcal{N}) = \frac{k}{n}e_n$。

请注意，选拔机制在不同维度上存在差异，尤其是：(i) 候选人是否经过预先选定（$m < n$），还是任何人都可以成为议事团的一员（$m = n$）；(ii) 输出的议事团是否具有预先规定的规模；以及 (iii) 每位代理人是否都与其所支持的议事团成员中的某一位存在直接联系（$\|f^{M_k}(\Gamma, \mathcal{C})\|_1 = n$），还是有些代理人会被虚拟代表（即由他们未必投票支持的人代表）（$\|f^{M_k}(\Gamma,\mathcal{C})\|_1 < n$）。我们分别将这些维度称为开放—封闭、灵活—刚性和直接—虚拟。上述选拔机制均位于所诱导的三维空间中的不同位置，因为其中一些机制在设计上允许更大的灵活性，或代表更多代理人。我们希望了解这些设计选择对理想公理的影响。反过来，这一由此产生的三维空间有助于比较不同的机制。我们预计，来自这一三维空间某个特定区域的机制，在满足我们某些公理方面表现得尤其出色（或尤其不佳）。

3 公理

我们聚焦于五项公理，每一项都体现了代表性的不同方面：比例性、多样性、单调性、忠实性和有效性。对于这些公理，我们既借鉴（计算）社会选择领域，也借鉴政治学。我们所描述的公理是阐明各种选拔机制所蕴含内容的第一步；这并不是假定这些要求是唯一重要的要求，或者它们是“最值得追求的要求”。例如，人们可能希望根据被选出的议事团所作决策的质量，或被选出的议事团的问责性，来研究选拔机制。然而，这超出了本文的范围。

ε–比例性。比例性刻画了代理人在代表议事团中的预期投票权重对其预期得票份额的反映有多“准确”。比例性对于实现描述性代表尤其理想（Black 1958；Brams 2008；Mansbridge 1999；Valsangiacomo2021；Zick 2013），也与政治学中此前针对不同选拔公式的比例性度量研究有关（Rae 1967）。[7] 我们提出一种 ε-比例性的概念，要求对于每位候选人，其归一化后的预期得票份额与归一化后的预期投票权重之差至多为 ε.。为定义这一概念，令

$$diff(\Gamma,\mathcal{C},M_k) = \max_{j\in[n]}\left|\frac{\mathbb{E}[V_j]}{\|\mathbb{E}[V]\|_1} - \frac{f^{M_k}(\Gamma,\mathcal{C})_j}{\|f^{M_k}(\Gamma,\mathcal{C})\|_1}\right|.$$

那么，ε-比例性要求 $\text{diff}(\Gamma,\mathcal{C},M_k) \le \varepsilon$。对于取决于候选人闭合集合的选拔机制，将 $\overline{\varepsilon^{M_k}} = \max_{\mathcal{C}\subset\mathcal{N}}\text{diff}(\Gamma, \mathcal{C}, M_k)$ 和 $\underline{\varepsilon^{M_k}} = \min_{\mathcal{C}\subset\mathcal{N}}\text{diff}(\Gamma,\mathcal{C},M_k)$ 分别定义为：在所有可能的候选人集合中，候选人的预期投票权重与其预期得票份额之间最大偏差的最大值和最小值。

为了给出 ε–比例性的一个例子，我们再次使用示例 1 中描述的设置。在表 1 中，我们给出了 ε 的值，对于这些值，每种选拔机制在示例 1 上都是 ε-比例的。我们看到，在这种情况下流动民主具有最小的 ε 值；而简单多数制未能将投票权重高效地分配给所选议事团。通过这一公理研究选拔机制的一个有趣收获是，我们看到抽签遴选的比例性与议事团的规模无关，包括抽签遴选规模等于人口规模（$k = n$）从而恢复直接民主的情况。

多样性 主要与审议阶段相关（Chamberlin and Courant 1983; Landemore 2020; Duarte et al. 2015），我们将多样性解释为要求所有意见都应出现在代表机构中。我们将其形式化为“如果某候选人的预期得票份额为正，则其预期投票权重也应为正”，即 $\mathbb{E}[V_j] > 0 \text{ implies } f^{M_k}(\Gamma,\mathcal{C})_j > 0 \text{ for all } j \in \mathcal{C}$。

单调性 这一基准在社会选择理论中是标准的（Moulin，1988）。设 Γ 和 Γ' 是一些代表矩阵。若在代表矩阵 Γ' 中，候选人 j 的预期得票份额大于在 Γ 中的预期得票份额，且预期

[6] 请注意，如果存在委托循环，循环中代理人的选票将会作废。因此，他们的投票权重设为零，这些代理人实际上会被忽略。

[7] 请注意，比例性可以定义为对选票、属性或偏好的描述性代表（Mansbridge 1999）。在此，我们仅对已表达选票的比例性进行模型化，而不涉及选民群体的任何其他特征。

	f^D	f^{F_1}	f^P	f^L	f^{S_k}
ε	0.16	[0.33, 0.86]	[0.13, 0.33]	0.06	0.16

Table 1: The minimum values of ε for $\varepsilon-$proportionality in Example 1 for each of the different selection mechanisms: direct democracy (D), first-past-the-post (F), proxy voting (P), liquid democracy (L), and sortition (S). The intervals denote the best and worst-case ε over all candidates sets. For sortition, the presented ϵ value holds for all sizes of the body $\mathbf{k} \in [\mathbf{n}]$.

得票份额对其他任何候选人都没有增加，那么 j的预期投票权重就会增加。也就是说，若 Γ 和 Γ' 满足以下条件：对于所有 $i \neq j$，$\mathbb{E}[V_j'] > \mathbb{E}[V_j]$ 且 $\mathbb{E}[V_i'] \leq \mathbb{E}[V_i]$，那么不等式 $f^{M_k}(\Gamma', \mathcal{C})_j \geq f^{M_k}(\Gamma, \mathcal{C})_j$ 应当成立。

Faithfulness This axiom ensures that candidates are not hurt by having a higher expected vote share. The axiom requires that if a candidate has a higher vote share than some other candidate, then they also have a higher expected voting weight as computed by the mechanism M, i.e., $\mathbb{E}[V_i] \geq \mathbb{E}[V_j]$ implies $f^{M_k}(\Gamma, \mathcal{C})_i \geq f^{M_k}(\Gamma, \mathcal{C})_j$ for all $i, j \in \mathcal{C}$.

$\gamma-$**effectiveness** Finally, *effectiveness* models potential deadlocks when no majoritarian coalition may come to an agreement. This benchmark measures the size of the smallest coalition needed to have majority support for some proposal. For a given mechanism M and candidate set $\mathcal{C}$, it is defined as the expected smallest number $\gamma_{\mathcal{C}}^{M_k}$ such that some coalition of $\gamma_{\mathcal{C}}^{M_k}$ representatives gather strictly more than half of the voting weight. For mechanisms that rely on a specified set of candidates, it would again be interesting to look at the worst and best-case scenarios for $\gamma_{\mathcal{C}}^{M_k}$ over all possible candidates set of fixed size.

4 Discussion

We have argued that there is a need for a more systematic comparison of different selection mechanisms within a unified framework to understand the trade-offs inherent to the selection mechanisms currently on the table to open democratic representation (Landemore 2020). Taking a first step in this direction, we have presented a simple model that allows the formulation of many different selection mechanisms together with axioms derived from political science and social choice theory that can be used to compare and assess these mechanisms.

我们不认为我们的模型和公理是最终的或详尽的，并且我们相信提出正确的问题已经是第一个研究挑战。然而，我们的研究产生了一些有趣的开放问题：这些机制中哪些总是满足多样性、单调性和忠实性？我们能否获得不同机制的ϵ-比例性或 γ-有效性的有意义界限？虽然似乎不太可能获得一般界限，但我们希望识别表示矩阵的特征可能对应于某些比例性和有效性值的保证。此外，获得比较陈述会很有趣，在这种意义在于，某一机制总能保证优于另一机制（至少在选择了"正确"的候选人集合时如此）。最后，人们也可能想探究候选人的数量以及所选候选人集合对我们的机制在公理性表现上的影响。更一般地说，若能识别出有助于提升公理性保证的选拔机制的一般特征，将会很有意义，这些特征或许可以在我们所讨论的开放—封闭、灵活—刚性以及直接—虚拟维度的基础上进一步拓展。

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